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A Dynamic Model of Semi-Rigid Connections for Seismic-Resistant Steel Structures

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ABSTRACT

In this paper we propose a mathematical model simulating the dynamic behaviour of a semi-rigid joint in order to evaluate its structural response. The joint is modelled as a discrete element with mass m, subjected to inertial forces and actions of connecting elements, considered as springs with unilateral reaction and linear elastic-perfectly plastic behaviour.

Utilizing a step by step procedure and a numerical simulation we can obtain the force-displacement diagrams, that permit to evaluate the resistance and ductility capacity of the joint, taking into account also the influence of damping forces.

INTRODUCTION

In the design of seismic-resistant steel structures, solutions that foreseen the realisation of semi-rigid connections instead of rigid ones. are more and more widespread. Such solutions, often used because of the easy carrying out from a technological point of view, turn out very effective when subjected to seismic forces, as both theoretical analyses, extensively available in the literature, and damage analysis of structures owing to an earthquake, show.

The presence of semi-rigid connections changes very much the seismic response as regard that one of structures with rigid joints. Semi-rigid connections, in fact, are the first elements in the structure showing an inelastic behaviour, as consequence, a right design of connection ductility makes possible to obtain better responses of structures in seismic areas. However, the analysis of seismic response, is very difficult, mostly because of the difficulties in the connection modelling. Several models are available in the literature, some of them are based on complex analytical relationships (Fléjou, 1994), others are defined on the basis of experimental results (Mazzolani, Faella and De Martino, 1984). At present it must point out the absence of design Codes, even if some indications in a such way come from seismic Eurocode (EC8, 1993) and from some seismic Codes in Japan and in USA (Astaneh-Asl, 1994).

In the present paper, with reference to semi-rigid connections, usually used in buildings construction, we propose a mathematical model that permits to evaluate theirs dynamic response under seismic forces. In particular, the model refers to connections of top-and-seat angle type, that are the most widespread in the current technical practice and enough representative of semi-rigid connections behaviour.

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In order to analyze the proposed model, the solution of dynamic equations in every field is carried out with a numerical procedure of step-by-step integration (La Tegola and Sarà, 1970, 1971).

A large analysis allows to determine the force-displacement law, the resistance capacity and the ductility of the connection, taking into account the presence of dissipative forces. Numerical results allow to put in evidence the influence of mechanical parameters on the dynamic behaviour of connections.

DYNAMIC MODEL OF SEMI-RIGID JOINTS

The proposed mathematical model refers to the scheme in Fig. 1. In such scheme the element of inertial mass m represents the system connected by the joint, the couples (m_1, S_1) and (m_2, S_2) the connecting model, on the right and the left side respectively, that the joint achieves. Connecting elements, carried out with flanges and bolts, are considered of unilateral type because of the bolts are supposed resistant only if stressed by tensile forces.

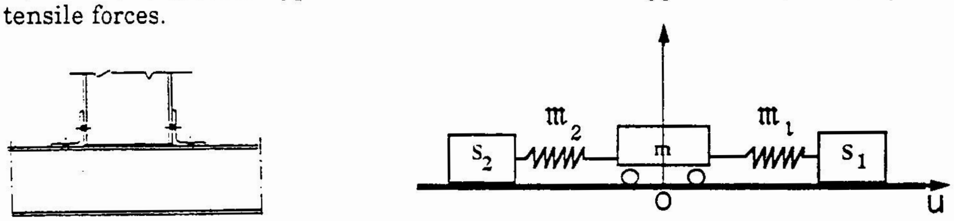


Fig. 1. Semi-rigid joint model

Referring to that above-mentioned, the mechanical behaviour of the joint is characterized by the consitutive law shown in Fig. 2. In every field the dynamic equilibrium equation is given by the following relationship

$$F_a + F_v + F_k = F_t \tag{1}$$

where $F_a = ma_m$ is the inertial force of the joint with mass m, $F_v = cv_m$ the dissipative force, $F_t = ma_t$ the external force, F_k the spring reaction, being a_m and v_m the

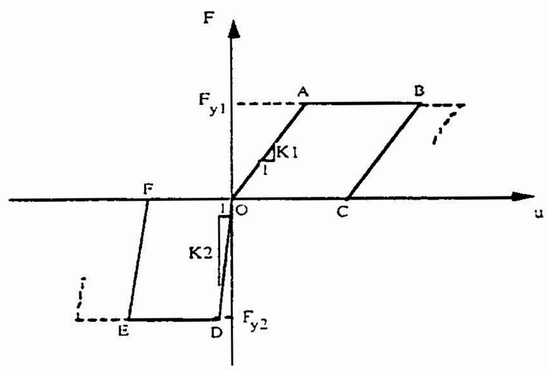


Fig. 2. Constitutive law of the model

acceleration and velocity of the mass m, c the viscosity, a_t the ground acceleration.

In order to carry out numerical solutions, for a better understanding of the influence of various parameters, it is necessary to use adimensional quantities. Then considering

$$\overline{u}_m = \frac{u_m}{|u_{yr}|};\quad \overline{v}_m = \frac{v_m}{|u_{yr}|} T_0 \sqrt{\frac{m}{K_r}};\quad \overline{u}_1 = \frac{u_1}{|u_{yr}|};\quad \overline{u}_2 = \frac{u_2}{|u_{yr}|};\quad \overline{u}_1^* = \frac{u_1^*}{|u_{yr}|};\quad \overline{u}_2^* = \frac{u_2^*}{|u_{yr}|};\quad \delta\overline{u}_1 = \frac{\delta u_1}{|u_{yr}|};\quad \delta\overline{u}_2 = \frac{\delta u_2}{|u_{yr}|}$$

$$\overline{a}_m = \frac{a_m}{|u_{yr}|} T_0^2 \frac{m}{K_r};\quad \overline{a}_t = \frac{a_t}{|u_{yr}|} \frac{m}{K_r};\quad \xi = \frac{c}{2m}\sqrt{\frac{m}{K_r}};\quad \overline{K}_1 = \frac{K_1}{K_r};\ \overline{K}_2 = \frac{K_2}{K_r};\quad \overline{F}_{y1} = \frac{F_{y1}}{|F_{yr}|};\ \overline{F}_{y2} = \frac{F_{y2}}{|F_{yr}|}$$

in which $K_r=\max(K_1,K_2)$, being K_1 and K_2 the stiffness of the resistant element on the right and on the left respectively; u_{yr} the first yielding displacement value of the spring having stiffness K_r, $F_{yr}=u_{yr}K_r$, $T_0=2\pi$, and u_1, u_2, δu_1, δu_2, u_1^*, u_2^* displacements, displacements increments and residual displacements respectively of S_1 and S_2 systems. Referring to that above-mentioned and to Fig. 2, for every field, the equation (1) can be expressed as follows, respecting the corresponding limits of validity.

Field OA: displacement of the mass m towards S_1; elastic behaviour of the spring m_1

In this field the behaviour of the spring m_1 is elastic while the spring m_2 is not connected to the mass m. Therefore, respecting following conditions

$$\overline{u}_m > 0;\quad \overline{v}_m > 0;\quad \overline{K}_1\left(\overline{u}_m - \overline{u}_1^*\right) \le \overline{F}_{y1};\quad \delta\overline{u}_1 = 0;\quad \delta\overline{u}_2 = 0$$

the equilibrium equation (1) is given by:

$$\frac{\overline{a}_m}{T_0^2} + 2\xi\frac{\overline{v}_m}{T_0} + \overline{K}_1\left(\overline{u}_m - \overline{u}_1^*\right) = \overline{a}_t$$

Field AB: displacement of the mass m towards S_1; plastic behaviour of the spring m_1

In this stage, got over the elastic limit, the behaviour of the spring m_1 is of plastic type while the spring m_2 is not connected to the mass m. Therefore we obtain

$$\overline{u}_m > 0;\quad \overline{v}_m > 0;\quad \overline{K}_1\left(\overline{u}_m - \overline{u}_1^*\right) \ge \overline{F}_{y1};\quad \overline{u}_1 = \overline{u}_1^* + \overline{u}_m - \frac{\overline{F}_{y1}}{\overline{K}_1};\quad \delta\overline{u}_2 = 0$$

and the equation (1) is given by

$$\frac{\overline{a}_m}{T_0^2} + 2\xi\frac{\overline{v}_m}{T_0} + \overline{F}_{y1} = \overline{a}_t$$

Field BC: displacement of the mass m towards S_2; elastic discharge of the spring m_1.

At the generic point B, a change of displacement direction of the mass m produces the elastic discharge of the spring m_1; the spring m_2 is not yet connected to the mass m. Resulting then

$$\overline{u}_m > 0;\quad \overline{v}_m < 0;\quad \overline{K}_1\left(\overline{u}_m - \overline{u}_1\right) \le \overline{F}_{y1};\quad \delta\overline{u}_1 = 0;\quad \delta\overline{u}_2 = 0$$

the equation (1) is the following:

$$\frac{\overline{a}_m}{T_0^2}+2\xi\frac{\overline{v}_m}{T_0}+\overline{K}_1\left(\overline{u}_m-\overline{u}_1\right)=\overline{a}_t$$

Field COF: displacement of the mass m towards S_2 without spring reaction

In this situation both springs are not connected to the mass m; for this reason the mass m slips in the displacement direction until the connection of the spring m_2 Conditions are:

$$\overline{u}_m>0;\quad \overline{v}_m<0;\quad \left|\overline{u}_m-\overline{u}_1\right|\geq\frac{\left|\overline{F}_{y1}\right|}{\overline{K}_1};\quad \delta\overline{u}_1=0;\quad \delta\overline{u}_2=0 \qquad \text{(field CO)}$$

$$\overline{u}_m<0;\quad \overline{v}_m<0;\quad \left(\overline{u}_m-\overline{u}_2^*\right)\geq 0;\quad \delta\overline{u}_1=0;\quad \delta\overline{u}_2=0 \qquad \text{(field OF)}$$

while the (1) is expressed as:

$$\frac{\overline{a}_m}{T_0^2}+2\xi\frac{\overline{v}_m}{T_0}=\overline{a}_t$$

Field OD: displacement of the mass m towards S_2; elastic behaviour of the spring m_2.

In this case the spring m_2 is connected to the mass m ; it presents an elastic behaviour while the springm_1 is not connected. Conditions are the following:

$$\overline{u}_m<0;\quad \overline{v}_m<0;\quad \overline{K}_2\left(\overline{u}_m-\overline{u}_2^*\right)\geq\overline{F}_{y2};\quad \delta\overline{u}_1=0;\quad \delta\overline{u}_2=0$$

and the equation (1) is expressed as:

$$\frac{\overline{a}_m}{T_0^2}+2\xi\frac{\overline{v}_m}{T_0}+\overline{K}_2\left(\overline{u}_m-\overline{u}_2^*\right)=\overline{a}_t$$

Field DE: displacement of the mass m towards S_2; elastic behaviour of the spring m_2

In this situation one obtain

$$\overline{u}_m<0;\quad \overline{v}_m<0;\quad \overline{K}_2\left(\overline{u}_m-\overline{u}_2^*\right)\leq\overline{F}_{y2};\quad \overline{u}_2=\overline{u}_2^*+\overline{u}_m-\frac{\overline{F}_{y2}}{\overline{K}_2};\quad \delta\overline{u}_1=0$$

and the equation (1) is expressed as:

$$\frac{\overline{a}_m}{T_0^2}+2\xi\frac{\overline{v}_m}{T_0}+\overline{F}_{y2}=\overline{a}_t$$

Field EF: displacement of the mass m towards S_1; elastic discharge of the spring m_2.

Conditions that describe this stage are:

$$\overline{u}_m<0;\quad \overline{v}_m>0;\quad \overline{K}_2\left(\overline{u}_m-\overline{u}_2\right)\geq\overline{F}_{y2};\quad \delta\overline{u}_1=0\ ;\quad \delta\overline{u}_2=0$$

and the equation (1) is expressed as:

$$\frac{\overline{a}_m}{T_0^2}+2\xi\frac{\overline{v}_m}{T_0}+\overline{K}_2\left(\overline{u}_m-\overline{u}_2\right)=\overline{a}_t$$

Field FOC: displacementoof the mass m towards S_1 without reaction of the springs

Considering following conditions:

$$\overline{u}_m < 0; \quad \overline{v}_m > 0; \quad |\overline{u}_m - \overline{u}_2| \geq \frac{|\overline{F}_{y2}|}{\overline{K}_2}; \quad \delta\overline{u}_1 = 0; \quad \delta\overline{u}_2 = 0 \qquad \text{(field FO)}$$

$$\overline{u}_m < 0; \quad \overline{v}_m > 0; \quad (\overline{u}_m - \overline{u}_1^*) \leq 0; \quad \delta\overline{u}_1 = 0; \quad \delta\overline{u}_2 = 0 \qquad \text{(field OC)}$$

and the equation (1) is expressed as:

$$\frac{\overline{a}_m}{T_0^2} + 2\xi\frac{\overline{v}_m}{T_0} = \overline{a}_t$$

With reference to the external force and initial conditions, the solution of the equation of dynamic equilibrium (1), is obtained by an incremental procedure defined by others authors (La Tegola and Sarà, 1970,1971). If $\tau^{(i)}=t^{(i)}-t^{(i-1)}$ represents the time interval corresponding to the i-th step and $\overline{\tau}_{(i)} = \tau_{(i)}/T_r$ its adimensional value, being $T_r = \min\left[T_1 = T_0\sqrt{\frac{m}{K_1}}; T_2 = T_0\sqrt{\frac{m}{K_2}}\right]$, one obtain the following relationships:

$$\frac{1}{T_0^2}\delta\overline{a}_m^{(i)} + 2\frac{\xi}{T_0}\delta\overline{v}_m^{(i)} + \overline{K}_j\delta\overline{u}_m^{(i)} = \delta\overline{a}_t^{(i)} \qquad j=1,2 \qquad (2)$$

in the case of elastic behaviour of the springs

$$\frac{1}{T_0^2}\delta\overline{a}_m^{(i)} + 2\frac{\xi}{T_0}\delta\overline{v}_m^{(i)} = \delta\overline{a}_t^{(i)} \qquad (3)$$

in the case of plastic behaviour of the springs and when the mass m slips.

Solutions of (2) and (3) are obtained using adequate displacement laws in the interval of time $\tau^{(i)}$; in particular solutions as

$$u_m = A_1 + A_2 t + A_3 t^2 + A_4 t^3$$

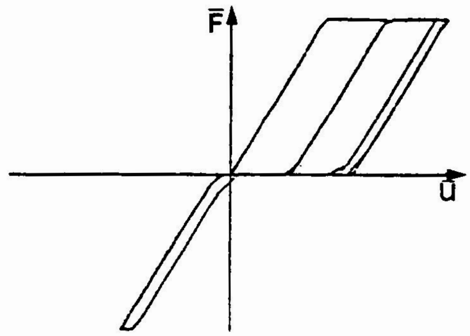
can be used to obtain nearly exact solutions; the A_i values, are determined considering limit conditions in each time interval. Using the incremental procedure above-mentioned (La Tegola and Sarà, 1970), if $u_m^{(i-1)}$, $v_m^{(i-1)}$, $a_m^{(i-1)}$ are initial values of tdisplacement, velocity and acceleration while $\delta u_m^{(i)}$ is the displacement variation during the time interval $\tau^{(i)}$, one obtain as final values, the quantities expressed as:

$$u_m^{(i)} = u_m^{(i-1)} + \delta u_m^{(i)}; \quad v_m^{(i)} = \frac{3\delta u_m^{(i)}}{\tau^{(i)}} - \frac{a_m^{(i)}}{2}\tau^{(i)} - 2v_m^{(i-1)}; \quad a_m^{(i)} = \frac{6\delta u_m^{(i)}}{\tau^{(i)2}} - 2a_m^{(i-1)} - \frac{6v_m^{(i-1)}}{\tau^{(i)}}$$

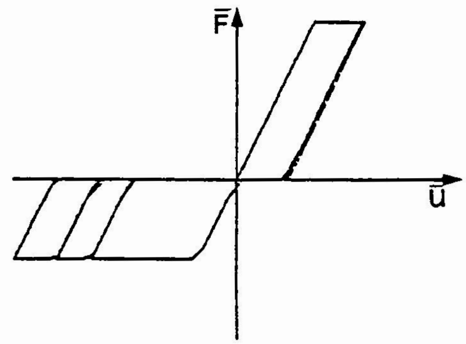
$$\delta a_m^{(i)} = \frac{6\delta u_m^{(i)}}{\tau^{(i)2}} - 3a_m^{(i-1)} - \frac{6v_m^{(i-1)}}{\tau^{(i)}}; \qquad \delta v_m^{(i)} = \frac{3\delta u_m^{(i)}}{\tau^{(i)2}} - \frac{1}{2}a_m^{(i-1)} - 3v_m^{(i-1)}$$

NUMERICAL APPLICATIONS

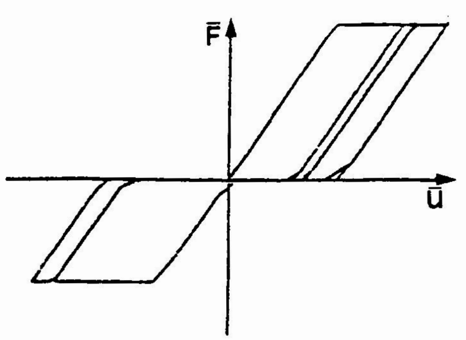
Using incremental relationships above described, a large numerical analysis, that allows to describe the dynamic behaviour of semi-rigid connections, is carried out varying mechanical caracteristics of the connection (K_1/K_2 ratio, damping coefficient ξ, F_{y1}/F_{y2} ratio). Figs.3 show force-displacement diagrams of the semi-rigid connection subjected to external forces of sinusoidal type $\bar{a}_t = \bar{A}_f \sin\left(T_0 \frac{\bar{t}}{\bar{T}}\right)$ being $\bar{A}_f = \frac{A_f}{|u_{yr}|}\frac{m}{K_r}$ with A_f amplitude of the external force and $\bar{T} = \frac{T_f}{T_r}$ being T_f the period of the external force.



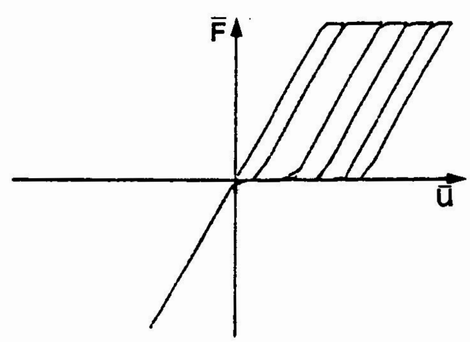
a (γ=1, α=1, $\bar{A}_f$=1.5, $\bar{T}$=3)



b (γ=2, α=1, $\bar{A}_f$=1.5, $\bar{T}$=3)



c (γ=1.5, α=1, $\bar{A}_f$=1.5, $\bar{T}$=3)



d (γ=2, α=1, $\bar{A}_f$=1.5, $\bar{T}$=2)

Fig. 3. Force -displacement diagrams of semi-rigid connections

Analysing diagrams one put in evidence in each cycle a behaviour of the connection caracterized, if the external forces exceed limit values, from plastic excurtions along the two directions with a relevant dissipation of energy (measured by the area included in each cycle). When the external force is lower than limit values the behaviour of the connection is elastic without dissipation of energy and, therefore, either situation can verify depending, obviously, from various parameters. Important parameters are the measurement of the external force values in comparison with the elastic limit of the connection and the period of the external force respect to the natural period of vibration of the connection. For this reason, known shape and intensity of the external force, the required ductility value can be defined varying rigidities and elastic limits of the above-mentioned parameters.

The Figs 4 and 5 show the diagrams μ- $\alpha^2=K_2/K_1$, being $\mu=u_{max}/u_y$ the adimensional value of the ductility of the connection, varying the damping coefficient ξ for $\gamma= F_{y1}/F_{y2}$ values equal to 1 and 2 respectively.

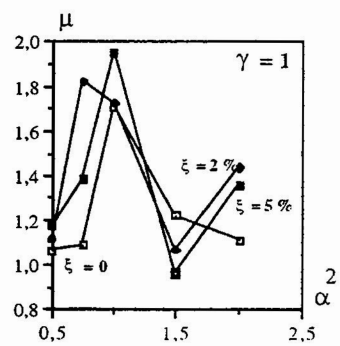


Fig.4. Diagrams μ-α^2 (γ=1)

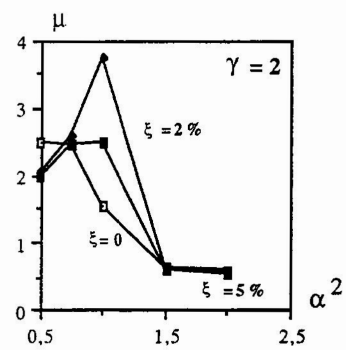


Fig.5. Diagrams μ-α^2 (γ=2)

Diagrams show the influence of dissipative forces on the ductility values of connections; in particular it is possibile, for each external force, to determine a range of values of rigidities of the connection corresponding to high values of ductility. Besides, it is possible to put in evidence the opportunity to design the semi-rigid connections varying the damping coefficient values and then dissipative forces, to guarantee a ductile behaviour of the connections.

CONCLUDING REMARKS

On the basis of obtained results one can be put in evidence that the proposed model represents adeguately the dynamic behaviour of semi-rigid connections avoiding high analytical difficulties and it is very effective to evaluate the influence of design parameters which optimize the dynamic behaviour with reference to the required ductility values.

REFERENCES

Astaneh-Asl, A. 1994. Seismic behaviour and design of steel semi-rigid structures. General Report. STESSA: International Workshop and Seminar on Behaviour of Steel Structures in Seismic Areas, Timisoara, Romania.

Eurocode 8. 1993. Structures in seismic regions. CEN, European Committee for Standardization, ENV.

Fléjou,J.L. 1994. Seismic effects on steel frames with semi-rigid joints: modeling, vibrations, damping. Proceedings of STESSA: International Workshop and Seminar on Behaviour of Steel Structures in Seismic Areas, Timisoara, Romania, pp. 6.41-6.50.

La Tegola, A., Sarà, G. 1970. Effetti della plasticità e della non linearità geometrica sulla risposta dinamica di un oscillatore elasto-plastico in regime di risonanza. Giornale del Genio Civile, Fasc. 4°, Roma, Italy.

La Tegola, A., Sarà, G. 1971. Sulle oscillazioni forzate di un oscillatore elasto-plastico a comportamento geometricamente non lineare. Proceedings of the First National Conference AIMETA, Udine, Italy.

Mazzolani, F.M., Faella, C., De Martino, A. 1984. Simulation of beam-to column joint behaviour under cyclic loads. Costruzioni Metalliche, n° 6.